The results obtained confirm the necessity for a deep analysis of the fluctuating structure to create reliable methods of computing the heat transfer.

NOTATION

C_f, local friction coefficient; $K = \frac{\nu}{U_\infty^2} \frac{dU_\infty}{dX}$, acceleration parameter; $R_{\overline{UV}}$ and $R_{\overline{V\theta}}$, correlation coefficients; $Re_T^{**} = U_\infty \delta_T^{**}/\nu$, Reynolds number; U, V, mean velocity components in the X and Y directions, m/sec; u', v', fluctuating velocity components, m/sec; St, Stanton number; X, Y, longitudinal and transverse coordinates, m; Y^+, "wall" coordinate; δ, boundary layer thickness, m; δ_T, thermal boundary layer thickness, m; δ^{**}, displacement thickness, m; δ_T^{**}, thickness of loss of energy, m; θ', temperature fluctuations, °K; ν, viscosity, m^2/sec. Subscripts: ∞, in the free stream.

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INFLUENCE OF PERMEABILITY OF AXISYMMETRIC SURFACES ON THEIR SEPARATION FLOW

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The separation flow of an ideal incompressible fluid around axisymmetric permeable surfaces is investigated on the basis of the model of a uniformly perforated surface.

1. Interest in the investigation of the separation flow around axisymmetric surfaces is related primarily to their utilization as different braking apparatus, of parachutes, say. General approaches to diagramming such separation flows within the framework of an ideal incompressible medium and effective numerical methods for their analysis on an electronic computer are proposed in [1]. On this basis the separation flow around axisymmetric surfaces of different shape, including during motion with acceleration, is studied [2]. It was here assumed in the computations that the streamlined body is impermeable.

As a rule, however, braking apparatus are fabricated from materials that are capable of transferring fluid particles under the effect of a pressure difference, i.e., are permeable. In addition to fabrics, among permeable surfaces are also different grids, perforated plates, shells, and other structures.

A periodic change in the velocities and pressures that is associated with the alternation of the impermeable and permeable surface sections is observed in its neighborhood during the flow around a permeable surface. The wake being formed behind it is shaped under the effect of both the external flow and the internal flow through the permeable surface. Exact modeling of the permeability phenomenon is a very complex hydrodynamic problem; consequently, analysis of the fluid flow through a permeable surface is performed expediently at the "hydraulic" level by means of the average flow characteristics by introducing a certain discontinuity surface. Such an approach to studying the aerodynamics of permeable surfaces was proposed

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by Rakhmatulin [3], who introduced the model of a uniformly permeable surface into the considerations. According to [3], the boundary condition on a uniformly permeable surface contains the average normal component of the relative velocity of the medium, the so-called penetration velocity V_i that is related to the magnitude of the pressure drop Δp acting at this point:

$$V_i = f(\Delta p). \tag{1}$$

The main disadvantage of the model of a uniformly permeable surface [3] is the absence of boundary conditions in the system, that are obtained from the general conservation laws on a discontinuuity, for the relationship taking account of hydraulic losses in the fluid flowing through the penetrable surface. This circumstance is felt principally in the determination of the hydrodynamic characteristics of the permeable surfaces and can result in contradictions to experiment. Thus, for example, an increase in the drag coefficient of a permeable plate was obtained in [4] as compared with an impermeable plate when a stationary incompressible fluid flowed around it. Results of experimental investigations indicate the reverse [5].

In the general case, the problem of analyzing the hydrodynamic characteristics of permeable surfaces with hydraulic losses in the average flow taken into account is sufficiently complex. However, if certain assumptions are introduced about the flow parameters in the fluid jet mixing domain that occur on the exterior side of the permeable surface, then the solution of the problem is simplified considerably.

Let us introduce the model of a uniformly perforated surface in the consideration, whereby we understand it to be an idealized surface to which the real permeable surface tends as the quantity of orifices increases while their total area or degree of permeability $\sigma = \Delta F/F$ is simultaneously conserved (ΔF is the total orifice area on a section of a permeable plate with area F).

We obtain a relationship connecting the hydrodynamic load acting on a uniformly perforated and uniformly permeable surface, for which we extract a control volume Q in the internal flow impinging on the section of the permeable surface F and we write the momentum equation for it:

$$\frac{\partial \mathbf{K}}{\partial t} + \int_Q \rho(\mathbf{V}\cdot\mathbf{n})\cdot\mathbf{V}ds = \int_Q \rho\mathbf{P}dQ + \int_S (p\mathbf{n})\,ds + \int_F \mathbf{R}ds. \tag{2}$$

The control surface S includes the stream surface and the normal section of the stream tube directly behind and ahead of the penetrable surface. The normal section ahead of the permeable surface is here selected in such a manner that during passage to the limit from the real penetrable surface to the uniformly perforated surface it approaches infinitely closely to this latter and the stream characteristics in this section would be independent of the specific structure of the perforations. It is easy to see that in this case the flow parameters in the section will tend to the corresponding flow parameters directly ahead of the uniformly permeable surface during passage to the limit.

To determine the parameters in the section directly behind the permeable surface, we make the assumption that the hydraulic losses are concentrated completely in the fluid jet mixing domain flowing out from the surface, and the pressure in this section (the base pressure) is determined primarily by the large-scale structure of the vortex wake rather than the local effects taking place in the jet mixing domain. Then taking account of the results of experiments [6], from which there follows that the fluid jet mixing domain is proportional to the characteristic size of the performations, and executing the passage to the limit to the uniformly perforated surface, we obtain that the pressure directly behind it equals the pressure behind the uniformly penetrable surface.

If the passage to the limit is now performed to the uniformly perforated surface in (2), then we obtain in projections on the normal to the surface F

$$R_n = \Delta p - \rho V_i^2 (1 - \alpha), \tag{3}$$

where R_n is the normal component of the hydrodynamic force (the average pressure drop Δp°) on the uniformly perforated surface.

Since $\alpha \approx 1/\sigma$, then the average pressure on the uniformly perforated surface turns out to be less than on the uniformly penetrable surface.

Let us note still another feature of the flow around permeable surfaces — the presence of tangential forces R_τ distributed over the surface, which we will characterize by the dimensionless coefficient $c_\tau = R_\tau/(^1/_2\rho V_e^2)$. In contrast to the normal hydrodynamic force R_n, the quantity R_τ is determined not only by the dependence (1) and the degree of permeability σ, but also by the form of the permeable surface (for instance, for infinitely thin surfaces with orifices $R_\tau = 0$). Consequently, a formula analogous to (3) can be obtained for c_τ only by introducing additional characteristics of the permeable surface. If the orifices in the permeable surface are not profiled in any special manner, then for an appropriate permeable surface thickness it can be considered that the fluid flows out of them along the normal to the surface, and therefore, the tangential component of the mean fluid velocity on the outer side of the surface equals zero ($V_{\tau-} = 0$). By projecting (2) on the direction of the tangent to the surface and performing the passage in the limit to the uniformly perforated surface, we obtain

$$R_\tau = \rho V_i V_{\tau+}, \tag{4}$$

where $V_{\tau+}$ is the tangential component of the mean fluid velocity on the inner side of the surface.

2. Taking account of the features noted above, we consider the mathematical formulation of the problem of the separation flow around a thin axisymmetric permeable surface by an ideal incompressible fluid flow. We assume that the flow is irrotational everywhere outside the surface and its wake. Then, the Laplace equation is valid for the velocity potential in this domain. Surfaces of tangential velocity discontinuity converge with the edges of the surface under consideration during its separation flow and the Chaplygin–Zhukovskii hypothesis on finiteness of the velocity should be satisfied on these edges. The boundary condition (1) is satisfied on the permeable surface itself, and the kinematic condition of continuity of the normal velocity component and the dynamic condition of continuity of the pressure, on the surfaces of tangential velocity discontinuity. Moreover, conditions at infinite distance from the body and the wake as well as given initial conditions are used in the problem under consideration.

The problem formulated above is reduced by the method of discrete vortices [1] to the solution of a system of algebraic equations in the desired vortex circulations at each nominal time. In the case of permeable surfaces, the system of equations is nonlinear here and iteration methods must be applied to solve it. As computations showed, good convergence of the iteration process can be obtained by using successive approximations with relaxation [7] in which the initial approximation in the penetration velocity V_i is determined at each time by the pressure difference in the preceding step.

The hydrodynamic load on the surface under consideration is determined by means of the vortex circulations found at each nominal time by using the Cauchy–Lagrange integral and taking account of the relationships (3) and (4). The wake structure is found from the condition of free-vortex motion together with the liquid particles. The stream characteristics (velocity and pressure fields, statistical characteristics, etc.) are calculated from the known vortex circulations and the accompanying wake structure.

3. Computations of the separation flow around axisymmetric surfaces of different shape (disc, hemisphere, sphere segment) with a different degree of permeability (σ = 0; 0.1; 0.2) were carried out on an electronic computer on the basis of the method expounded above. The dependence of the penetration velocity on the pressure drop was assumed quadratic, i.e.,

$$\overline{V}_i = k_w \operatorname{sign}(\overline{\Delta p^0}) \sqrt{|\overline{\Delta p^0}|}, \tag{5}$$

where $\overline{V}_i = V_i/V_e$ is the dimensionless penetration velocity, and $\overline{\Delta p^0} = \Delta p^0/(1/2\rho V_e^2)$ is the dimensionless pressure drop. Values of k_w and α corresponding to σ = 0.1 and 0.2 were taken equal to k_w = 0.11, 0.25 and α = 10, 5.

Depending on the nature of the motion, the axisymmetric surfaces were replaced by a different quantity of discrete annular vortices ($10 \le N \le 100$) in the computations. The total length of the axisymmetric surface generator was taken as the characteristic dimension D; here the computational spacing in the dimensionless time was $\Delta\overline{t} = 1/(2N + 1)$ $\left(\overline{t} = \frac{1}{D}\int_0^t V_e(t_1)\,dt_1\right.$ is the dimensionless time).

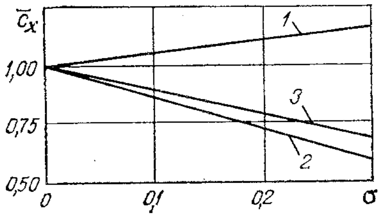

Fig. 1. Dependence of the magnitude of the relative disc drag on the degree of penetrability σ: 1) computation by the model of a uniformly permeable surface; 2) computation by the model of a uniformly perforated surface; 3) experimental data [5].

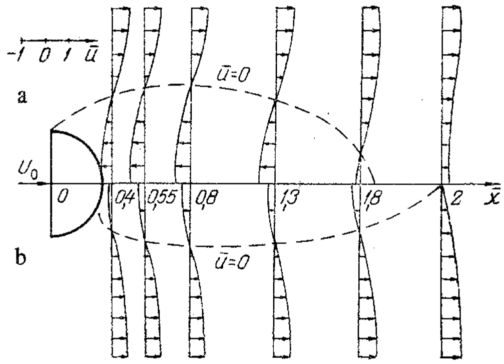

Fig. 2. Mean values of the longitudinal velocities: a) impermeable surface (σ = 0); b) permeable surface (σ = 0.2).

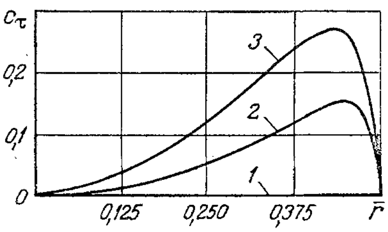

Fig. 3

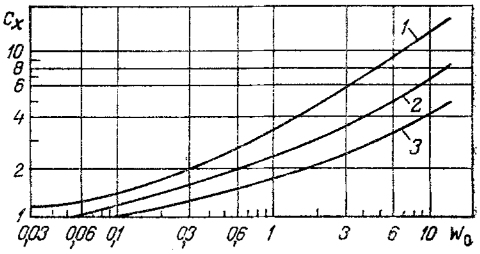

Fig. 4

Fig. 3. Distribution of the dimensionless tangential force over the disc surface ($\bar{t}$ = 10): 1) σ = 0; 2) 0.1; 3) 0.2.

Fig. 4. On the influence of acceleration and flat disc permeability on its drag: 1) σ = 0; 2) 0.1; 3) 0.2.

Computational and experimental dependences on the degree of penetrability are shown in Fig. 1 for the relative drag coefficient $\bar{c}_x$ of a disc, as determined from the formula $\bar{c}_x = c_x(\sigma)/c_x(\sigma = 0)$, where $c_x(\sigma)$ and $c_x(\sigma = 0)$ are the drag coefficients of the permeable and impermeable discs, respectively. As is seen from the figure, the results of computations by means of the uniformly perforated surface model are in good agreement with the experimental data [5]. At the same time, the model of a uniformly permeable surface yields a result contradicting experiment.

Diagrams of the longitudinal velocities $\bar{u}$ of the average flow in the wake behind a hemisphere are shown to scale in the sections $\bar{x} = x/D$ = const in Fig. 2 (x is the axial coordinate). The average was taken after termination of the flow formation transient during hemisphere motion according to the law

$$V_e(\bar{t}) = \begin{cases} 0, & \bar{t} < 0, \\ U_0, & \bar{t} \geqslant 0. \end{cases}$$

Lines of zero longitudinal velocity ($\bar{u} = 0$), the boundaries of the reverse flow domains, are superposed by dashes. As is seen, the reverse flow domain is shifted downstream behind the permeable hemisphere, its extent increases, and the reverse flow attenuates (longitudinal velocities diminish). An analogous pattern is observed in the wake and behind other permeable surfaces.

Figure 3 contains data on the distribution of the dimensionless tangential force characterized by the coefficient c_τ over the surface of the permeable disc. It is seen that the tangential forces are distributed nonuniformly over the surface and reach the maximal magnitude near the edge of the disc. Moreover, in contrast to the normal component of the force, which diminishes as the degree of permeability σ increases, the tangential component of the hydrodynamic force acting on the disc increases and can be commensurate with the normal component.

Dependences of the disc drag coefficient c_x on the dimensionless acceleration W are shown in Fig. 4 for different degrees of disc permeability. The motion law had the form

$$W(\bar{t}) = \begin{cases} 0, & \bar{t} < 0, \\ W_0, & \bar{t} \geqslant 0. \end{cases}$$

Here $W = (D/V_e^2)(dV_e/dt)$, where V_e and dV_e/dt are the instantaneous values of the velocity and acceleration.

NOTATION

V_i, penetration velocity; σ, degree of permeability; k_w, permeability coefficient; α, Boussinesq coefficient; **V**, fluid velocity vector; ρ, density; Δp, Δp°, pressure drops on the uniformly permeable and uniformly perforated surfaces, respectively; Q, control volume; S, control surface; K, momentum vector; **P**, volume force vector; **R**, vector of the distributed hydrodynamic forces acting on the permeable surface; V_e and dV_e/dt, velocity and acceleration of the surface motion; c_τ, dimensionless tangential force coefficient; c_x, drag coefficient; $\bar{c}_x$, relative drag coefficient; t, time; $\bar{t}$, dimensionless time; $\Delta\bar{t}$, computational time spacing; W, dimensionless acceleration; D, disc diameter; $\bar{u} = u/V_e$, longitudinal component of the dimensionless fluid velocity; N, quantity of discrete vortices; n, τ, normal and tangent to the surface. The superscripts "+" and "–" refer to different sides of the streamlined surface.

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